

# Answers to exam-style questions

## Option D

1 ✓ = 1 mark

- 1 a i Luminosity is the total power radiated by a star. ✓  
 ii Apparent brightness is the power received per unit area. ✓  
 b They all fuse hydrogen into helium. ✓  
 c i Using the mass-luminosity relation  $\frac{L_A}{L_\odot} = \left(\frac{M_A}{M_\odot}\right)^{3.5}$  ✓

$$\text{Hence } L_A = L_\odot \left(\frac{M_A}{M_\odot}\right)^{3.5} = L_\odot \times 6.7^{3.5} = 778 \times 3.9 \times 10^{26} = 3.04 \times 10^{29} \approx 3.0 \times 10^{29} \text{ W } \checkmark$$

$$\text{ii From } b = \frac{L}{4\pi d^2} \text{ we find } d = \sqrt{\frac{L}{4\pi b}} = \sqrt{\frac{3.04 \times 10^{29}}{4\pi \times 1.7 \times 10^{-8}}} = 1.19 \times 10^{18} \text{ m} = \frac{1.19 \times 10^{18}}{3.09 \times 10^{16}} \text{ pc} = 38.5 \approx 38 \text{ pc } \checkmark$$

$$\text{Hence } p = \frac{1}{d} = \frac{1}{38.5} = 0.0260 \approx 0.026'' \checkmark$$

$$\text{iii } \frac{L_A}{L_\odot} = 778 = \frac{\sigma 4\pi R_A^2 T_A^4}{\sigma 4\pi R_\odot^2 T_\odot^4} = \frac{R_A^2 T_A^4}{R_\odot^2 T_\odot^4} \checkmark$$

$$778 = \frac{R_A^2}{R_\odot^2} \times 2.6^4 \checkmark$$

$$\text{Hence } R_A = R_\odot \frac{778}{2.6^4} \approx 17 R_\odot \checkmark$$

- d i The parallax method measures the position of a star two times six months apart. ✓  
 The shift of the angular position of the star relative to the background of the distance stars. ✓  
 Allows measurement of the parallax angle which is the angle subtended by the earth's orbit radius at the star. ✓  
 The distance in pc is the reciprocal of the parallax angle in arc seconds. ✓  
 ii Yes because it is larger than the limit of 0.01 arc seconds. ✓
- 2 a Light reaching the Earth must go through the outer layers of the star. ✓  
 Photons whose energy corresponds to differences in energy between energy levels of the atoms of the star may be absorbed and so will be missing in the received light. ✓  
 Because the energy level differences are specific atoms determination of the chemical composition is then possible. ✓
- b i Type O stars are very hot and most of the hydrogen is ionised. ✓  
 Hence hydrogen cannot absorb any photons. ✓  
 ii An M type star is relatively cool so that hydrogen atoms are mostly in their ground state. ✓  
 And these can only absorb ultraviolet photons not visible light photons. ✓
- c The surface temperature/its magnetic field/its rotational speed. ✓
- 3 a The surface of the star periodically expands and contracts. ✓  
 It expands because radiation ionises helium atoms in the star's outer layers and the released electrons heat up the star expanding it. ✓  
 When most of the helium is ionised, radiation leaves the star so the star cools and contracts. ✓
- b There is a relation between the average luminosity of the star and the period of variation of the luminosity. ✓  
 So measuring the period gives the luminosity. ✓  
 Measuring the (average) apparent brightness (by observing the star over a period of days) allows determination of the distance. ✓  
 To measure the distance to a galaxy it must be determined whether a specific Cepheid star belongs to that galaxy. ✓

- c The average apparent brightness is estimated to be  $2.8 \times 10^{-8} \text{ W m}^{-2}$ . ✓  
 A period of 55 days corresponds to an average luminosity of about 20000 solar luminosities. ✓

$$\text{And so } d = \sqrt{\frac{L}{4\pi b}} = \sqrt{\frac{20000 \times 3.9 \times 10^{26}}{4\pi \times 2.8 \times 10^{-8}}} \approx 5 \times 10^{18} \text{ m} \quad \checkmark$$

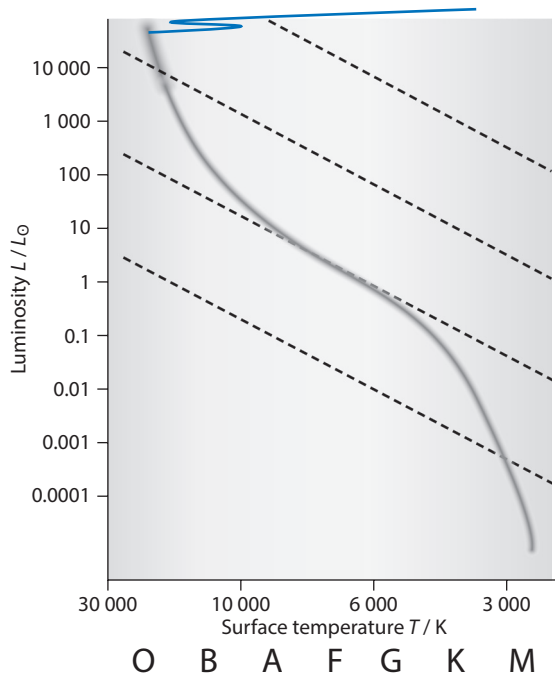
- 4 a i Using the mass-luminosity relation  $\frac{L_A}{L_\odot} = \left(\frac{M_A}{M_\odot}\right)^3$ . ✓

$$\text{Hence } L_A = L_\odot \left(\frac{M_A}{M_\odot}\right)^{3.5} = L_\odot \times 20^{3.5} = 3.6 \times 10^4 \times 3.9 \times 10^{26} = 1.4 \times 10^{31} \text{ W} \quad \checkmark$$

$$\text{ii } \frac{L_A}{L_\odot} = 3.6 \times 10^4 = \frac{\sigma 4\pi R^2 T^4}{\sigma 4\pi R_\odot^2 T_\odot^4} = 1.2^2 \times \frac{T^4}{T_\odot^4} \quad \checkmark$$

$$\text{Hence } \frac{T}{T_\odot} = \sqrt[4]{\frac{3.6 \times 10^4}{1.2^2}} = 12.6 \approx 13 \quad \checkmark$$

- b i The surface temperature decreases. ✓  
 And the radius increases. ✓  
 ii A type II supernova is the explosion of a massive star after it has entered the red supergiant phase while a type Ia supernova involves mass accretion onto a white dwarf. ✓  
 So in this case we have a type II supernova. ✓  
 iii The star will be a neutron star. ✓  
 In which the neutron degeneracy pressure. ✓  
 Balances the gravitational pressure in the star. ✓  
 c Blue line as shown starting approximately at the correct point. ✓



- 5 a i Strip from top left diagonally to bottom right. ✓  
 ii Region in lower left. ✓  
 iii Region to the right above main sequence. ✓  
 iv Region joining main sequence to red giants. ✓

b i  $\frac{L_X}{L_Y} = \frac{100}{0.01} = 10^4 = \frac{\sigma 4\pi R_X^2 T_X^4}{\sigma 4\pi R_Y^2 T_Y^4} = \frac{R_X^2}{R_Y^2} \quad \checkmark$   
 Hence  $\frac{R_X}{R_Y} = 10^2 \quad \checkmark$

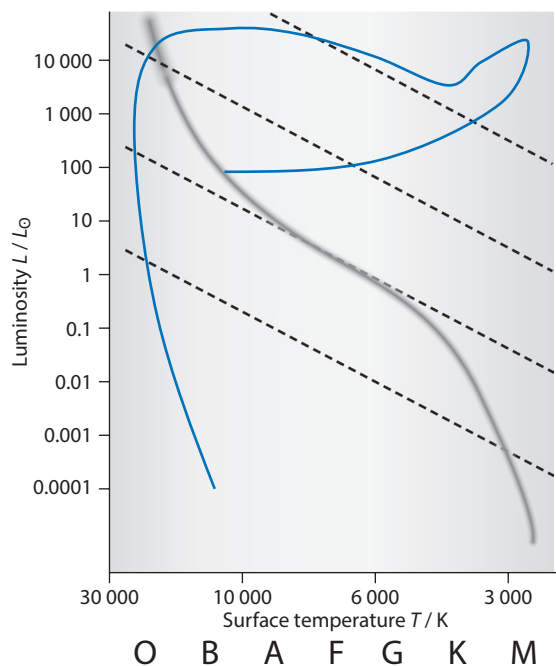
$$\text{ii } \frac{L_X}{L_Z} = 1 = \frac{\sigma 4\pi R_X^2 T_X^4}{\sigma 4\pi R_Z^2 T_Z^4} = \frac{R_X^2}{R_Z^2} \times \left(\frac{12000}{3000}\right)^4 = \frac{R_X^2}{R_Z^2} \times 256 \quad \checkmark$$

$$\text{Hence } \frac{R_X}{R_Z} = \sqrt{\frac{1}{256}} = \frac{1}{16} \quad \checkmark$$

$$\text{c } \text{Using the mass-luminosity relation } \frac{L_X}{L_\odot} = 100 = \left(\frac{M_X}{M_\odot}\right)^{3.5} \quad \checkmark$$

$$\text{Hence } \frac{M_X}{M_\odot} = 100^{1/3.5} = 3.7 \quad \checkmark$$

d i Line similar to blue line on HR diagram.  $\checkmark$



ii Electron degeneracy pressure.  $\checkmark$

Balances the gravitational pressure in the star.  $\checkmark$

iii It has to be less than the Chandrasekhar limit of 1.4 solar mass.  $\checkmark$

6 a Distant galaxies appear to move away from us with a speed that is proportional to their distance.  $\checkmark$

b The received wavelength.  $\checkmark$

Is larger than the wavelength at emission.  $\checkmark$

c On a large scale, the space between galaxies stretches.  $\checkmark$

Thus the wavelength of light emitted from a distant galaxy also stretches so it is larger at reception.  $\checkmark$

$$\text{d i } z = \frac{\Delta\lambda}{\lambda_0} = \frac{780 - 656}{656} = 0.189 \quad \checkmark$$

$$z = \frac{v}{c} \Rightarrow v = 5.67 \times 10^7 \approx 5.7 \times 10^7 \text{ m s}^{-1} \quad \checkmark$$

$$\text{ii } z = \frac{R}{R_0} - 1 = 0.189, \text{ i.e. } \frac{R}{R_0} = 1.189 \quad \checkmark$$

$$\frac{R_0}{R} = 0.84 \quad \checkmark$$

$$\text{iii } \text{The data give a Hubble value constant of: } v = Hd \Rightarrow H = \frac{5.67 \times 10^7}{920 \times 10^6 \times 3.09 \times 10^{16}} = 1.99 \times 10^{-18} \text{ s}^{-1} \quad \checkmark$$

$$\text{Hence } T = \frac{1}{H} = \frac{1}{1.99 \times 10^{-18}} = 5.0 \times 10^{17} \text{ s} \left( = \frac{5.0 \times 10^{17}}{365 \times 24 \times 3600} = 1.6 \times 10^{10} \text{ year} \right) \quad \checkmark$$

- e To see whether the universe accelerates or decelerates in its expansion distant objects of large redshift had to be investigated. ✓

In order to establish the relation between distance and redshift. ✓

Type Ia supernovae were chosen because their peak luminosity is known and hence the distance could be established by measuring the apparent brightness. ✓

- 7 a According to the hot big bang model the early universe contained radiation at very high temperature. ✓  
As the universe expanded it cooled and the peak wavelength shifted to large microwave wavelengths with a black body spectrum. ✓  
Which is what is being observed. ✓

b i  $z = \frac{R}{R_0} - 1 = \frac{T_0}{T} - 1$  ✓

$$z = \frac{3 \times 10^3}{2.7} - 1 = 1110 \quad \checkmark$$

ii  $\frac{R}{R_0} - 1 = 1110$  ✓

Hence  $\frac{R_0}{R} = 9 \times 10^{-4}$  ✓

- 8 a The result follows from  $\frac{GMm}{r^2} = m \frac{v^2}{r}$ . ✓

- b With  $M = kr$  the result in a becomes  $v = \sqrt{\frac{Gkr}{r}} = \sqrt{Gk}$  a constant. ✓

- c i The rotation curve becomes flat at large distances from the galactic centre. ✓

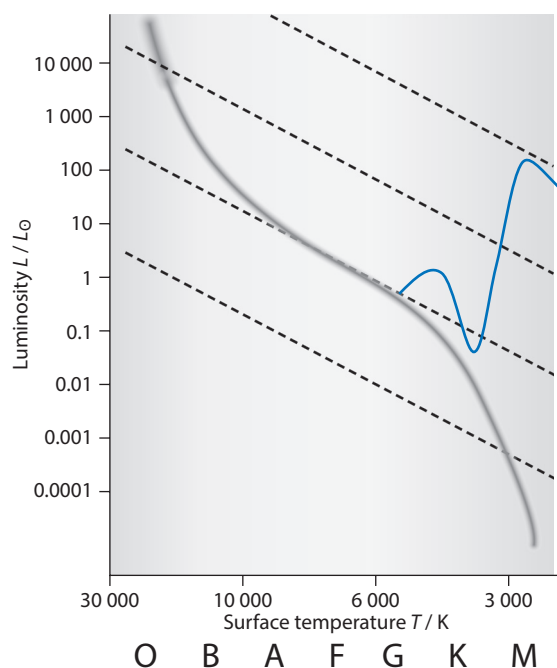
This is consistent with a mass distribution as in b. ✓

In other words with substantial mass far from the galactic centre. ✓

- ii Small planets/brown dwarfs/black dwarfs. ✓

Neutrinos/exotic particle predicted by supersymmetry. ✓

- 9 a The Jeans criterion states that cloud of gas will begin to collapse under its own gravitation. ✓  
When the gravitational potential energy of the cloud exceeds the total random kinetic energy of its particles. ✓
- b Blue line as shown. ✓



c i Use  $M = Nm$  to eliminate  $N$  so that  $\frac{GM^2}{R} \approx \frac{3}{2} \frac{M}{m} kT$  or  $\frac{GM}{R} \approx \frac{3}{2} \frac{kT}{m}$  ✓

Now eliminate the mass  $M$  through:  $M = \rho V = \rho \frac{4\pi R^3}{3}$  so that  $\frac{G\rho 4\pi R^3}{3R} \approx \frac{3}{2} \frac{kT}{m}$  ✓

Cancelling powers of  $R$  and simplifying gives the result  $\left( R^2 \approx \frac{9}{8\pi} \frac{kT}{mG\rho} \right)$  ✓

ii  $R^2 \approx \frac{9kT}{8\pi G\rho m} = \frac{9 \times 1.38 \times 10^{-23} \times 100}{8\pi \times 6.67 \times 10^{-11} \times 1.8 \times 10^{-19} \times 2.0 \times 10^{-27}}$  ✓

$R \approx 1.4 \times 10^{17} \text{ m}$  ✓

10 a i  $v = H_0 R$  ✓

ii  $E = \frac{1}{2} mv^2 - \frac{GMm}{R}$  ✓

$E = \frac{1}{2} m H_0^2 R^2 - \frac{G\rho 4\pi R^3 m}{3R} = \frac{1}{2} m H_0^2 R^2 - \frac{G\rho 4\pi R^2 m}{3}$  ✓

Factoring gives the result.

iii To escape to infinity the total energy must be zero. ✓

This means that  $H_0^2 - \frac{G\rho 4\pi}{3} = 0$ . ✓

From which the result follows.

iv  $\rho = \frac{3 \times \left( \frac{68 \times 10^3}{10^6 \times 3.09 \times 10^{16}} \right)^2}{8\pi \times 6.67 \times 10^{-11}}$  ✓

$\rho \approx 9 \times 10^{-27} \text{ kg m}^{-3}$  ✓

b In cosmological models with matter density parameters  $\rho_m$  and dark energy density  $\rho_\Lambda$  the significance of the critical density is that when  $\rho_m + \rho_\Lambda = \rho_c$ . ✓

The geometry of the universe is flat, i.e. it has zero curvature. ✓

11 a The CMB is very isotropic which means that we observe the same spectrum in every direction. ✓

However there are small deviations from perfect isotropy in the sense that, in different directions, the temperature deviates from the mean temperature of  $T = 2.723 \text{ K}$  by very small amounts (of order  $\frac{\Delta T}{T} \approx 10^{-5}$ ). ✓

b These deviations are significant because fluctuations in temperature imply fluctuations in density. ✓

And these are required if structures are to develop in the universe. ✓

They are also significant because the magnitude of the fluctuations depends on the geometry of the universe. ✓

Hence study of the fluctuations places limits on the geometry of the universe. ✓

12 a Elements are produced by nuclear fusion. ✓

And nuclear fusion becomes energetically impossible past the peak of the binding energy per nucleon curve which is at iron. ✓

b These are produced mainly by neutron capture. ✓

Nuclei may absorb neutrons which are abundant in a supernova. ✓

As these decay by beta decay. ✓

Nuclei with higher atomic number than iron are produced. ✓

c The CNO cycle requires the fusion of nuclei of carbon, nitrogen and oxygen and since these have a high atomic number the Coulomb barrier that must be overcome is larger than that for hydrogen and helium. ✓

And this requires higher temperatures that are found in the more massive stars. ✓